Determination of the neutron star mass-radii relation using narrow-band gravitational wave detector

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Abstract

The direct detection of gravitational waves will provide valuable astrophysical information about many celestial objects. The most promising sources of gravitational waves are neutron stars and black holes. These objects emit waves in a very wide spectrum of frequencies determined by their quasi-normal modes oscillations. In this work we are concerned with the information we can extract from f and p_I-modes when a candidate leaves its signature in the resonant mass detectors ALLEGRO, EXPLORER, NAUTILUS, MiniGrail and SCHENBERG. Using the empirical equations, that relate the gravitational wave frequency and damping time with the mass and radii of the source, we have calculated the radii of the stars for a given interval of masses M in the range of frequencies that include the bandwidth of all resonant mass detectors. With these values we obtain diagrams of mass-radii for different frequencies that allowed to determine the better candidates to future detection taking in account the compactness of the source. Finally, to determine which are the models of compact stars that emit gravitational waves in the frequency band of the mass resonant detectors, we compare the mass-radii diagrams obtained by different neutron stars sequences from several relativistic hadronic equations of state (GM1, GM3, TM1, NL3) and quark matter equations of state (NJL, MTI bag model). We verify that quark stars obtained from MIT bag model with bag constant equal to 170 MeV and quark of matter in color-superconductivity phase are the best candidates for mass resonant detectors.

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I. INTRODUCTION

The detection of gravitational waves (GWs) will have many implications in Physics and Astrophysics. Besides the confirmation of the general relativity theory, it will allow the investigation of several astrophysical phenomena, such as the existence of black holes and the mass and abundance of neutron stars, thus opening new scientific frontiers. The most promising sources for detection of GWs are neutron stars and black holes. These objects emit waves in a very wide spectrum of frequencies determined by their quasi-normal modes oscillations [1].

With the goal to analyze the possibility of a future detection of the quasi-normal modes of compact stars by resonant mass detectors (RMDs), we focalize our attention in the region of the spectrum in the range at 0.8-3.4 kHz, which is the operation region of the antennas: ALLEGRO, EXPLORER, NAUTILUS, AURIGA [2], SCHENBERG, and MiniGrail. In particular we will work with the frequency band of the spherical detectors SCHENBERG and MiniGrail (2.8-3.4 kHz) [3, 4].

The SCHENBERG is the second spherical detector ever built in the world and the first equipped with a set of parametric transducers, which is installed at the Physics Institute of the University of Sao Paulo (at Sao Paulo city, Brazil). It has undergone its first test run in September 8, 2006, with three transducers operational. Recent information on the present status of this detector can be found in reference [5]. It is worth stressing also that among all known GW detectors, the spherical ones are the only one capable to determine the direction of the incoming wave [6, 7].

In this paper we present an extension of the work made by Marranghello [1], where the authors show results for a restricted band of frequency that include the SCHENBERG and MiniGrail bandwidth. In the present work we analyze the case of a possible future detection made by all resonant antennas and compare the mass and radius range obtained from the frequency bands of the GWs modes to the mass and radius calculated with several relativistic equations of state (EoS) models.

In section 2 we introduce the f and p_I-mode, in section 3 we show the mass-radii diagrams for the f and p_I modes and compare them with the ones obtained with different relativistic EoS models, in section 4 we introduce the damping time of the f-mode and its mass-radii diagram and finally, in section 5, we made the last considerations.

II. THE QUASI-NORMAL MODES: F AND P$_I$-MODES

The neutron stars have a rich spectrum of frequencies because the fluid perturbation oscillates in many different modes. From the GW point of view the most important quasi-normal modes are the fundamental mode of the fluid oscillation (f-mode), the first pressure mode (p$_I$-mode), the first GW mode (w$_I$-mode) [8] and the r-modes that, under certain circumstances, can be an important source of GWs [9].

In this work we concentrate in the f and p$_I$-modes. The fundamental mode can be described by the density distribution inside the star, while the p-mode is the pressure restoration force. In reference [10], the authors have obtained an empirical formulae for the frequencies of these two modes as a function of the mass and radius using a wide sample of equations of state:

$$\nu_f = (0.79 \pm 0.09) + (33 \pm 2)\sqrt{\frac{M}{R^3}}, \tag{1}$$

$$\nu_p = \frac{1}{M}\left[(-1.5 \pm 0.8) + (79 \pm 4)\frac{M}{R}\right], \tag{2}$$

where the mass and the radii are given in km (remember that $M_\odot \approx 1.477$ km), while ν_f and ν_p are given in kHz. Using the empirical relations (1) and (2), we have calculated the radii R of the stars for a given interval of masses M in the range of frequencies that include the bandwidth (0.8-3.4 kHz) of all RMDs in operation. In Table (1) we can see the resonant frequencies of these detectors.

TABLE I: Frequency band of the RMDs in operantion in the world.

Antena	Location	Freq.(Hz)	Type
ALLEGRO	Baton Rouge	890-920	Bar
EXPLORER	CERN	895-920	Bar
NAUTILUS	Frascai	905-925	Bar
AURIGA	Legnaro	850-930	Bar
SCHENBERG	São Paulo	3100-3300	Spherical
MiniGrail	Leiden	2800-3000	Spherical

Through these relations we have obtained diagrams for p$_I$ and f-modes that relate GW frequency with masses and radii of the sources. These diagrams allow us to determine the

better candidates for a future detection by resonant antennas from the compactness of the star. We can see in figure (1) and (2) the f and p_I-mode's mass-radii diagrams where the different gray scale identify the different frequencies.

III. COMPARISON OF THE MASS-RADIUS DIAGRAMS WITH THE ONES OBTAINED BY RELATIVISTIC EOS MODELS

To determine what relativistic models of compact stars emit GW in the frequency bands of the RMDs we compare the diagrams of the relations (1) and (2) with some neutron stars masses and radii sequences obtained by different relativistic models that generate several equations of state for hadronic matter such as models NP, NPH, NPHQ with and without isovector-scalar δ [11], namely

- the models with parameters set GM1, GM3, NL3, TM1 [12, 13]

and some for quark strange matter as

- Nambu-Jona-Lasínio model (NJL) [14], color-flavor locked phase (CFL) [15] and the MIT bag model to different values of the bag constant [16] and hybrid star EoS.

Relativistic hadronic models have been widely used in order to describe nuclear matter, finite nuclei, stellar matter properties, and recently in the high temperature regime produced in heavy ion collisions [17]. Many variations of the well known quantum hadrodynamic model [18] have been developed and used along the last decades. Some of them rely on density dependent couplings between the baryons and the mesons [19, 20, 21, 22, 23, 24, 25] while others use constant couplings [26, 27, 28]. Still another possibility of including density dependence on the lagrangian density is through derivative couplings among mesons and baryons [29, 30, 31] or the coupling of the mediator mesons among themselves [32, 33, 34]. The relativistic model couplings are adjusted in order to fit expected nuclei properties such as binding energy, saturation density, compressibility and energy symmetry at saturation density, particle energy levels, etc. These same relativistic models are extrapolated to higher densities as in stellar matter and the results obtained for the neutron star masses and radii are quite good in comparison with the astronomical observations. In the case of bare quark stars, the strange matter inside the star is usually describe by the MIT bag model, a Fermi

gas of free quarks with a vacuum energy known as the bag constant, or by chiral models like the Nambu-Jona-Lasínio (NJL) model that has a dynamical chiral symmetry breaking mechanism that originates mass for the quarks. Recently, the possibility that quarks can be paired at high densities and be in a color superconductive phase has originated new quark matter equations of state, that depending on the pairing interaction, can be quite stiff and produce large stars masses and radii [35, 36, 37]. The main feature of a quark star, since they are bound by the strong force and not by gravity, is that they are more compact and have smaller mass to radius ratio than a neutron star. As we will see, it is this fact that strange stars can have small radii will explain the high frequency GWs modes produced by this type of stars.

We can see in the diagrams (1) and (2) that the frequency band of the RMDs are on the dark region, where it is expected that GWs generate from less compact neutron stars in both diagrams. This fact shows the impossibility of a future detection, by cylindrical antennas, of relativistic neutron star candidates emitting gravitational wave on p_I or f-mode. However, we can see on the diagrams that for the spherical detectors bandwidth, MiniGrail and Schenberg, have some candidates near their resonant frequencies. The most probable source would correspond to a very compact object with radius smaller than 10 km. The models that fulfill this condition are models of strange quark stars, as preview in [1]. This fact is confirmed when we compare with the compact star sequence generated from the MIT bag model (with bag constant $B^{1/4} = 170$ MeV), NJL model and CFL of quark matter. On the other hand the p_I-mode would only be expected to come from less compact neutron stars.

IV. THE DAMPING TIME

How can we distinguish the f-mode in a putative detection? And how to determine the mass and radii of the star? The damping time is the response for these questions [1]. In [10] the authors obtained an empirical relation for the f-mode damping time as function of the radius and mass, described by:

$$\tau_f = \frac{R^4}{cM^3}\left[(8.7 \pm 0.2)\cdot 10^{-2} + (-0.271 \pm 0.009)\frac{M}{R}\right]^{-1}. \qquad (3)$$

Even though the RMDs can not determine the damping time properties with small errors, we use the empirical relation (3) to calculate the damping time given the intervals of radii and mass (R, M) obtained with relation (1). We can get a new mass-radius diagram, but doing a distinction in the damping time. We compare the diagram with models of quark stars, CFL and MIT bag model with bag constant equal to 170 MeV. Results obtained can be seen in figure (3). Through these results we can estimate the masses and radii of the stars solving the inverse problem, as show in [9].

V. SUMMARY

RMDs bandwidth are on the spectrum regions with a few (or none) neutron stars candidates emiting GWs through their f and p_I-modes. However, the spherical detectors are on a region where the f-modes of very compact objects can be detected. All sequences of neutron stars described by quark matter models are on the region near to MiniGrail and Schenberg bandwidth, but the MIT bag model with bag constant equal to 170 MeV and CFL of the quark matter with the bag constant 200 MeV and the gap 100 MeV are the best candidates for these detectors, as we can see in figures (1) and (3). On the other hand the detection of the f and p_I-modes of neutron stars by bar detectors is unlikely, because their bandwidth is located in low frequencies.

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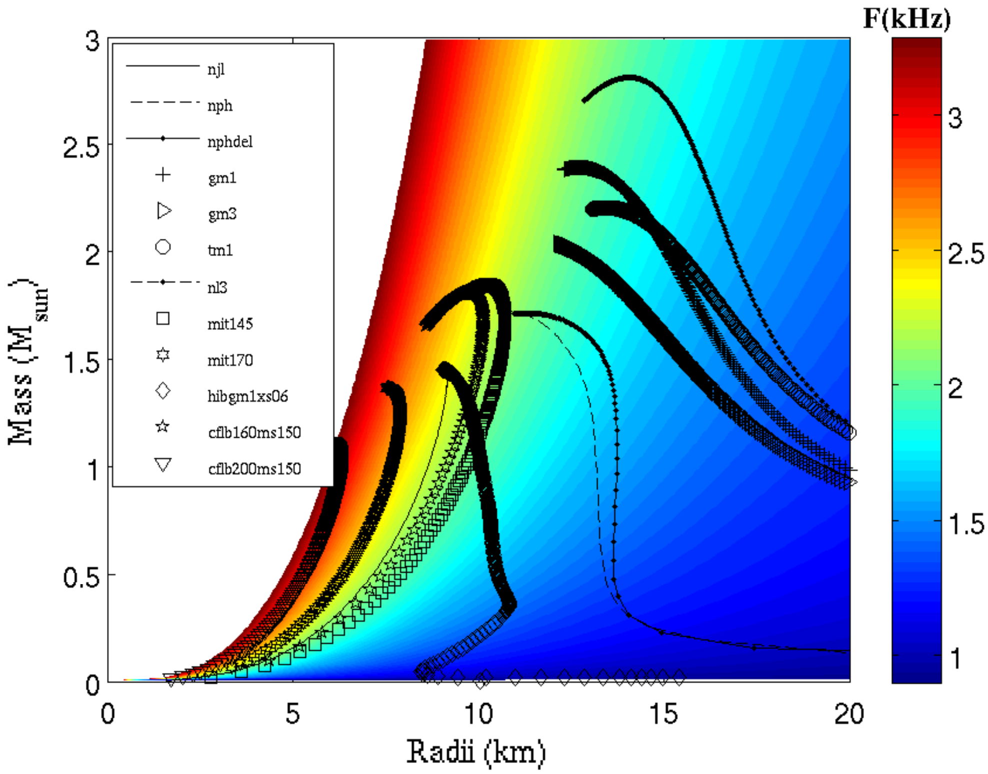

FIG. 1: Mass-radii diagram of the empirical relation 1 abtained by Benhar *et al* for f_I-mode. We compare with some models of relativistic EoS. The differents gray scale identify the different frequencies.

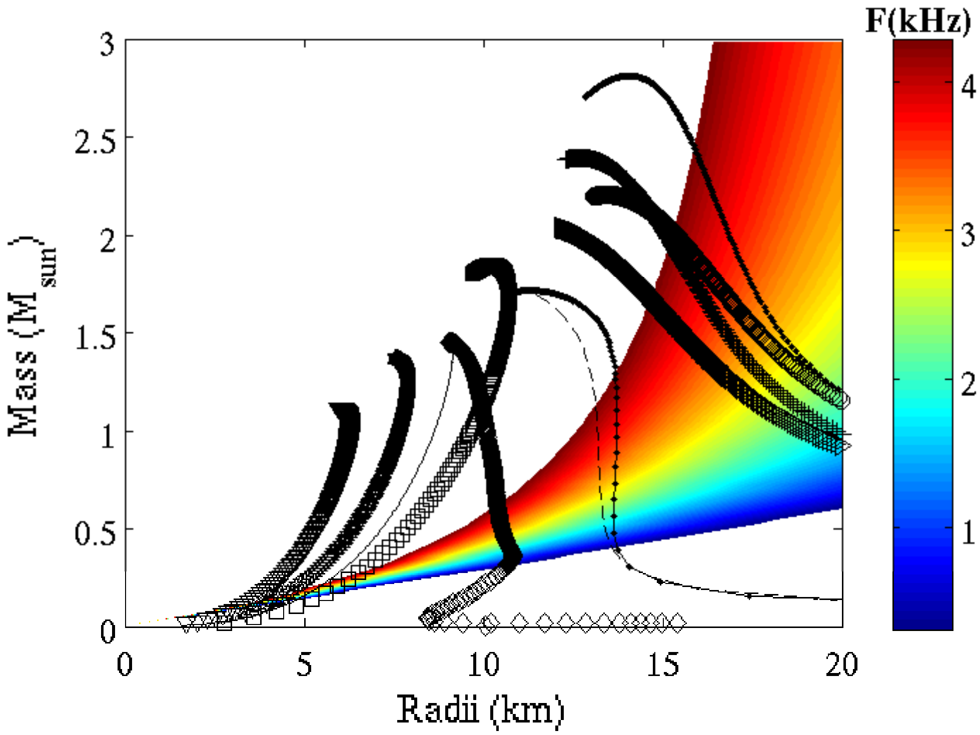

FIG. 2: Mass-radii diagram of the empirical relations 2 abtained by Benhar *et al* for p_I-mode. We compare with some models of relativistic EoS. The differents gray scale identify the different frequencies.

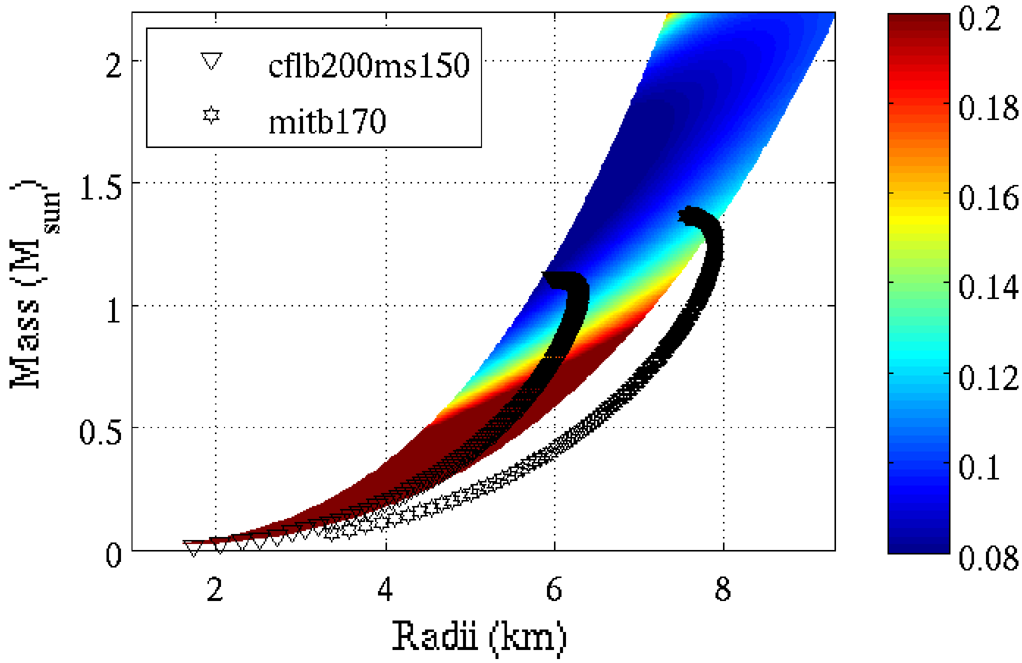

FIG. 3: Mass-radii diagram of the empirical relations 2 abtained by Benhar *et al* for damping time of the f_I-mode. We compare the diagram with models that discribe quark stars, CFL of the quark matter with bag constant 200 MeV and gap 100 MeV and MIT bag model with bag constant equal to 170 MeV. The differents gray scale identify the different damping time.